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SOLUTIONS OF EXERCISES.

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A homogeneous heavy rod is hung from a fixed point by elastic threads of given length fastened at its extremities. Find the position of equilibrium.

[*W. M. Thornton.*]

SOLUTION.

Assuming the rod to be of uniform section, the position of equilibrium is the same as if W , the weight of the rod, was concentrated at its middle point, through which passes the vertical through the point of suspension.

Let l and l'' be the given lengths of the strings, and μ , μ' such weights as would stretch them to double their original lengths. Let dl and dl'' be their respective elongations, and θ the angle between the strings in the position of equilibrium.

T and T' being the tensions in the strings, we have from statical relations

$$T^2 + T'^2 + 2TT' \cos \theta = W^2; \quad (1)$$

also

$$T/T' = (l + dl)/(l + dl'').$$

By Hooke's law,

$$T = \mu dl/l; \quad T' = \mu' dl''/l''. \quad (2)$$

Hence

$$T/T' = \mu' dl'/\mu l dl'' = (l + dl)/(l'' + dl''). \quad (3)$$

From (1) and (2) result

$$\left(\frac{\mu' dl}{\mu l dl''} \right)^2 + 1 + 2 \frac{\mu dl''}{\mu' l' dl} \cos \theta = \left(\frac{Wl''}{\mu' dl''} \right)^2;$$

and from the geometry of the figure we get

$$\left(\frac{l + dl}{l' + dl''} \right)^2 + 1 + 2 \frac{l + dl}{l' + dl''} \cos \theta = \left(\frac{2x}{l' + dl''} \right)^2,$$

where x is the distance of the middle of the rod below the point of suspension.

These two equations give, by virtue of relation (3),

$$dl'' = \frac{Wl''^2}{2\mu' x - Wl''}, \quad \text{and also} \quad dl = \frac{Wl^2}{2\mu x - Wl}.$$

Therefore the stretched lengths of the strings are

$$\frac{l'}{1 - \frac{Wl'}{2\mu'x}} \quad \text{and} \quad \frac{l}{1 - \frac{Wl}{2\mu x}},$$

where x is to be found from the geometrical relation

$$\left(\frac{l'}{1 - \frac{Wl'}{2\mu'x}} \right)^2 + \left(\frac{l}{1 - \frac{Wl}{2\mu x}} \right)^2 = 2(x^2 + a^2),$$

in which a is half the length of the given rod. This equation for x is of the sixth degree, which is best solved when μ and μ' are large, and therefore the elongations $Wl/2\mu$ and $Wl'/2\mu'$ are small compared with x , by using the median of the unstretched triangle as an approximate value of x and substituting successively in the first member of the equation. [W. H. Echols.]

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$f(x)$ and $\varphi(e^x)$ being rational algebraic functions of x and e^x , respectively, show that

$$\int f(x) \varphi(e^x) dx$$

depends upon integrals of three forms,

$$\int \frac{du}{\log u}, \quad \int \theta^m \tan \theta d\theta, \quad \int \frac{du}{(u+c)^r \log u}.$$

[R. A. Harris.]

SOLUTION.

There are, evidently, four forms to be considered; viz.,

$$\int x^m e^{ux} dx, \quad \int \frac{e^{ux}}{(x+a)^m} dx, \quad \int \frac{x^m}{(e^x+b)^n} dx, \quad \int \frac{dx}{(x+a)^m (e^x+b)^n}.$$

1. The first of these integrals can be found by successive integrations by parts.
2. The second, when treated in like manner, will be found to depend upon

$$\int \frac{e^{ux}}{x+a} dx.$$

This, in turn, depends upon the well-known transcendent

$$\int \frac{du}{\log u}.$$

3. Let us transform the expansion $\frac{x^m}{(e^x + b)^n} dx$ by letting $z = e^x$; then its integration may be made dependent upon integrals of the form

$$\int \frac{(\log z)^m}{z + b} dz,$$

as follows from considering the partial fractions into which

$$\frac{(\log z)^m}{z(z + b)^n} dz \left(= \frac{x^m}{(e^x + b)^n} dx \right)$$

may be decomposed. Next, let $u = z/b$; then the above integral will depend upon

$$\int \frac{(\log u)^m}{u + 1} du.$$

If we let $\cos \theta + i \sin \theta = u^{\frac{1}{b}}$ it is not difficult to show that this integral depends upon

$$\int \theta^m \tan \theta d\theta.$$

When x and b are real and the latter is essentially negative, we should write $u = z/b$, $\cos \theta - i \sin \theta = u^{\frac{1}{b}}$, thus making the integral in question depend upon

$$\int \theta^m \cot \theta d\theta,$$

which is, obviously, reducible to integrals of the form just written. It may be observed that when mod. θ lies between 0 and 1, mod. u lies between $1/e^2$ and e^2 .

4. If

$$\frac{dx}{(x + a)^m (e^x + b)^u}$$

be integrated by parts $m - 1$ times, the integration will be seen to depend upon integrals of the form

$$\int \frac{e^{qx}}{(e^x + b)^r} \cdot \frac{dx}{x + a},$$

where $q \leq m - 1$, $r \leq m + u - 1$. This, in turn, depends upon

$$\int \frac{du}{(u + c)^r \log u}.$$

[R. A. Harris.]